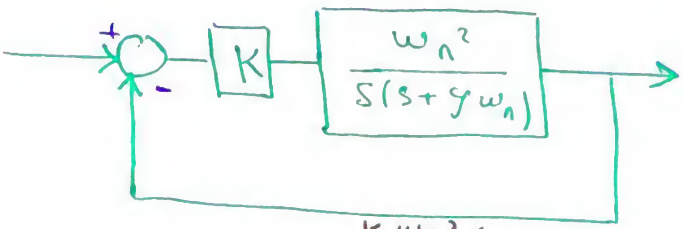
Control

15/10/2015

سكشن 3 　 م. شهاب 　 الخميس

Root Locus



$$C.L.T.F = \frac{\dfrac{K\omega_n^2}{s(s+2\zeta\omega_n)}}{1+\dfrac{K\omega_n^2}{s(s+2\zeta\omega_n)}}$$

$$= \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+K\omega_n^2}$$

Example

$$O.L.T.F = \frac{K}{s(s+2)}$$

$$C.L.T.F = \frac{K}{s^2+2s+K}$$

$K=1 \Rightarrow (s+1)^2$

$$K=0.5 \Rightarrow s_{1,2} = \frac{-2 \pm \sqrt{4-2}}{2}$$

$$= -1 \pm \frac{1}{\sqrt{2}}$$

Example:

$$O.L.T.F = \frac{K\,Z(s)}{P(s)}$$

$$C.L.T.F = \frac{K\,Z(s)/P(s)}{1+K\,Z(s)/P(s)}$$

$$= \frac{K\,Z(s)}{P(s)+K\,Z(s)}$$

at $K=0$

C.L. poles = O.L. poles

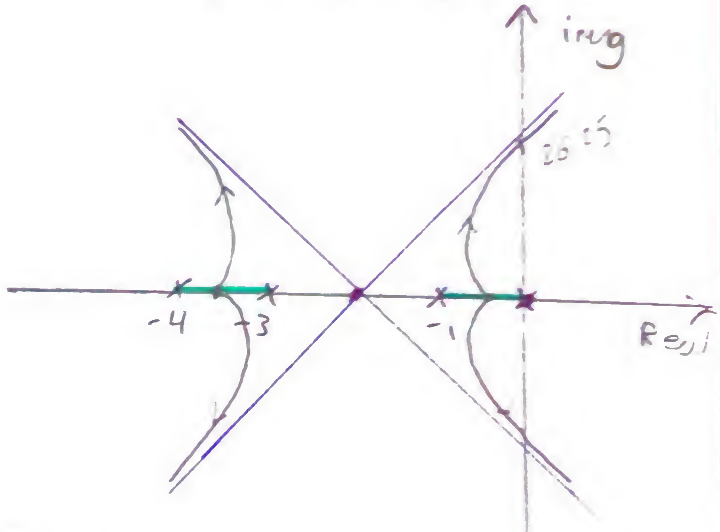
at $K=\infty$

O.L. poles = O.L. zeros

Q: $GH(s) = \dfrac{K}{s(s+1)(s+3)(s+4)}$

① on the sketch, determine (plot) open loop poles and zeros

$n_p = \checkmark$, $n_z = \checkmark$



② Determine the parts of the real axis that belong to root locus

The parts of the real axis located at the left of odd number of o.L. poles and zeros belong to the root locus

⇒ Turn over

③ Asymptotic lines

$\# = n_p - n_z$

$\sigma = \dfrac{\Sigma P - \Sigma Z}{n_p - n_z}$

$\theta = \dfrac{(2l+1) * 180}{n_p - n_z}$ $\quad l = 0, 1, 2$

$\# = 4$

$\sigma = \dfrac{-1-3-4}{4} = -2$

$\theta = 45, 135, 225, 315$

$= \pm 45, \pm 135$

④ Breaking points

$K = \dfrac{-1}{GH(s)}$

$\dfrac{dK}{ds} = 0$

$\Rightarrow$ Solve for S

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for higher order Systems

$K = \dfrac{-1}{GH(s)}$

S	--	-3.5	--	--	--
K					

شوف أكبر قيمة لل K

⑤ Range of K for stability

* use Routh Array

$1 + GH(s) = 0$

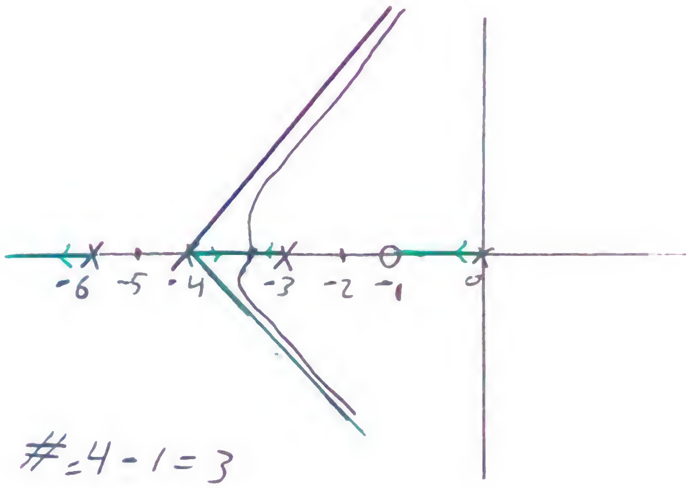
$S(S+1)(S+3)(S+4) + K = 0$

$s^4 + 8s^3 + 19s^2 + 12s + K = 0$

s^4	1	19	K
s^3	8	12	
s^2	17.5	K	
s^1	$\dfrac{12-8K}{17.5}$		
s^0	$K > 0$		

$K = 26.25 \Longleftarrow$ جرب !

Q: $GH(s) = \dfrac{K(s+1)}{s(s+6)(s+3)(s+4)}$



$\# = 4 - 1 = 3$

$\sigma = \dfrac{-6-3-4+1}{3} = -4$

$\theta = \dfrac{(2l+1) * 180}{3} = 60, 180, -60$

④ Breaking Points

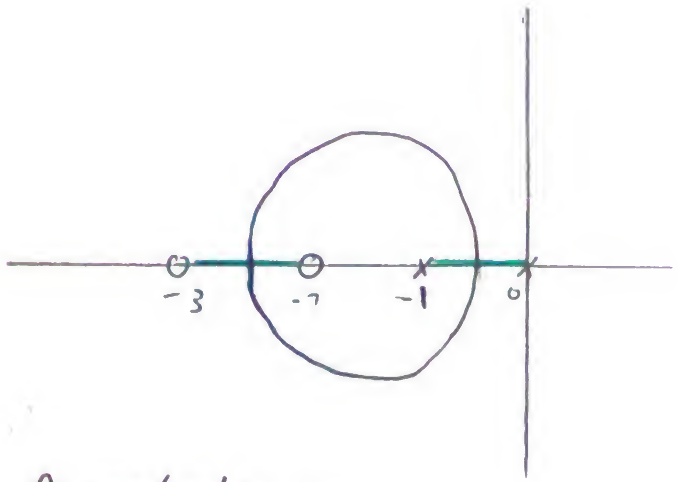
S	-3.5
K	

$K = -\dfrac{s(s+3)(s+4)(s+6)}{s+1}$

⑤ Range of K

Routh (do it yourself)

Q: $GH(s) = \dfrac{K(s+2)(s+3)}{s(s+1)}$



Asymptotes = 0

- Breaking points (Maximum)

out b: -1

S	--	-0.5	--
K			

$K = \dfrac{-1}{GH(s)} = \dfrac{-s(s+1)}{(s+2)(s+3)}$

in -2 : -3 Minimum

S		-2.5	
K			

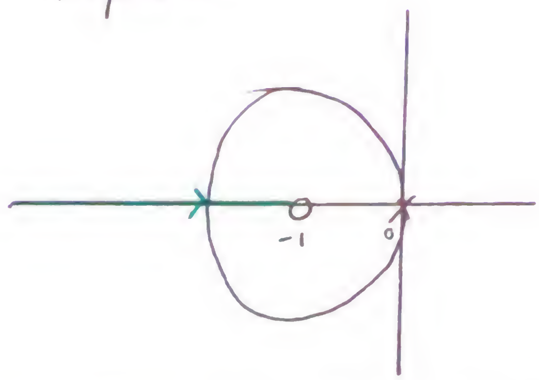
Report (optional)

Prove that the root Locus must be Circle

hint: Angle Condition

Q: $GH(s) = \dfrac{K(s+1)}{s^2}$

report




```
num=[ 1 5 6 ];
den = [ 1 1 0 ];
G = tf (num, den);
rlocus (G)
```

MatLab